

Multiple Magnetic Topological Phases in the van der Waals Crystal $\text{Mn}(\text{Bi,Sb})_4\text{Se}_7$

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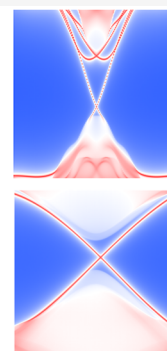
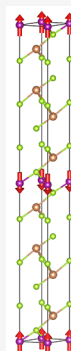


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Supporting Information

ABSTRACT: Magnetic topological materials have drawn markedly attention recently due to the strong coupling of their novel topological properties and magnetic configurations. In particular, the $\text{MnBi}_2\text{Te}_4/(\text{Bi}_2\text{Te}_3)_n$ family highlights the researches of multiple magnetic topological materials. Via first-principles calculations, we predict that $\text{Mn}(\text{Bi, Sb})_4\text{Se}_7$, the close relatives of $\text{MnBi}_2\text{Te}_4/(\text{Bi}_2\text{Te}_3)_n$ family, are topological nontrivial in both antiferromagnetic and ferromagnetic configurations. In the antiferromagnetic ground state, $\text{Mn}(\text{Bi, Sb})_4\text{Se}_7$ are simultaneously topological insulators and axion insulators. Massless Dirac surface states emerge on the surfaces parallel to the z axis. In ferromagnetic phases, they are axion insulators. Particularly, when the magnetization direction is along the x axis, they are also topological crystalline insulators. Mirror-symmetry-protected gapless surface states exist on the mirror-invariant surfaces. Hence, the behaviors of surface states are strongly dependent on the magnetization directions and surface orientations. Our work provides more opportunities for the study of magnetic topological physics.



**MnBi_4Se_7
and
 MnSb_4Se_7 :
Tunable
Topological
Surface
States**

As one of the most fantastic areas of modern condensed matter physics, topological quantum materials have attracted a great deal of interest.¹ Since the early concepts of topological insulators were conceived,^{2–4} a series of topological materials of various classes have been proposed theoretically and realized experimentally, such as topological crystalline insulators, Dirac semimetals, Weyl semimetals, and nodal line semimetals.^{5–7} The nontrivial band topology guarantees the symmetry-protected surface or hinge states and quasi-particle energy dispersions in these materials. As a result, novel physical properties, such as quantum spin Hall effect, quantum anomalous Hall effect, can be realized on these material platforms.^{8,9} Thus, topological states in condensed matter are promising for future dissipationless spintronics devices and quantum computations.¹⁰

The early discovery of topological materials mostly focuses on the nonmagnetic systems. Typical examples are the Bi_2Se_3 family topological insulators,¹¹ the SnTe class topological crystalline insulators,¹² the Cd_3As_2 and Na_3Bi Dirac semimetals,^{13,14} and the TaAs family Weyl semimetals.^{15,16} For the recently developed higher-order band topology, the early condensed matter realization was also in the nonmagnetic platforms.^{17,18} This is partially ascribed to the fact that the theoretical calculations for nonmagnetic materials is much more uncomplicated than the magnetic counterparts. Furthermore, the topological quantum chemistry (TQC) and symmetry indicators (SIs) also significantly facilitate the diagnosis of topological materials.^{19,20} In the recent days, Vergniory et al.²¹ diagnosed all topological bands of all nonmagnetic stoichiometric materials in the Inorganic Crystal Structure Database. It seems that the discovery of topological

electronic states in the nonmagnetic materials is tending to be well established.

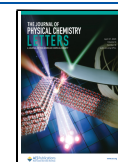
As for the study of magnetic topological phases, unlike the nonmagnetic counterparts, is still obstructed by lack of sufficient material realizations. The concept model of antiferromagnetic topological insulators was first proposed in 2010.²² However, the material realization of this topological phase was not achieved until 2019, when MnBi_2Te_4 was uncovered.²³ Further study of MnBi_2Te_4 confirmed that it is simultaneously an axion insulator in antiferromagnetic state while a Weyl semimetal when tuned to ferromagnetic state.²⁴ Quantum anomalous Hall effect is also observed in this material.²⁵ Furthermore, such topological phases, are all strongly coupled to specific magnetic configurations, and thus can be easily tuned and applied to the spintronics devices.^{26–28} Such novel performance is impossible to be realized in nonmagnetic materials. Following studies unveil that the $\text{MnBi}_2\text{Te}_4/(\text{Bi}_2\text{Te}_3)_n$ ($n = 1, 2, 3$) are similar tunable magnetic topological materials.^{29–35} Topological superconductivity and Majorana edge states were also predicted based on this material family.^{36,37}

As a close relative to $\text{MnBi}_2\text{Te}_4/(\text{Bi}_2\text{Te}_3)_n$ family, $\text{MnSb}_2\text{Te}_4/(\text{Sb}_2\text{Te}_3)_n$ family remains relatively less explored.

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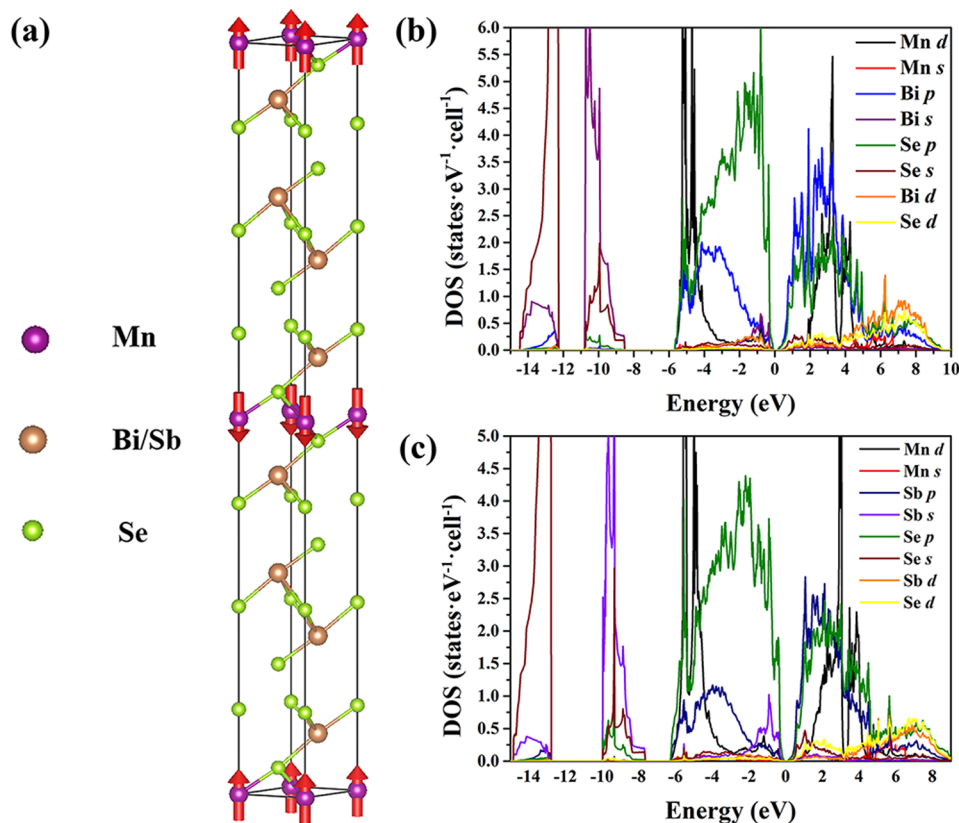


Figure 1. Structures and PDOS of Mn(Bi, Sb)₄Se₇. (a) The crystal structure, (b) the PDOS of MnBi₄Se₇, and (c) the PDOS of MnSb₄Se₇.

The Weyl semimetal state and highly tunable magnetic properties are confirmed in MnSb₂Te₄.^{38–42} Another highlighted material, MnSb₄Te₇, was successfully synthesized by Guo et al.,⁴³ and a huge anomalous Hall conductivity (AHC) was observed in experiments. It is also expected to be an antiferromagnetic topological insulator and an axion insulator simultaneously in its magnetic ground state. When tuned to the ferromagnetic state with a small external magnetic field, it is an axion insulator regardless of the magnetization direction.^{43,44} A ferromagnetic ground state is expected under strain or charge doping.⁴⁵ In Mn₂(Bi, Sb)₂Te₅ compounds, an antiferromagnetic topological insulator state and a ferromagnetic axion insulator or Weyl semimetal state are predicted.^{46–48} The MnBi₂Se₄ was discovered to be a high-temperature topological insulator both theoretically and experimentally.^{49,50} In view of these emerging exotic phenomena, it is obvious that more magnetic topological materials with similar structures are desired for the study of topological physics and spintronic applications.

In this work, we predict that MnBi₄Se₇ and MnSb₄Se₇ are magnetic topological material candidates via first-principles calculations. In their interlayer antiferromagnetic (A-AFM) ground state, they are simultaneously topological insulators and axion insulators with massless Dirac Fermion dispersion emerging on the surfaces parallel to *z* direction. When turned to ferromagnetic states, they are axion insulators. Furthermore, topological crystalline insulator phases simultaneously emerge when they are magnetized along the *x* orientation. In this magnetic configuration, mirror-symmetry-protected gapless surface states are expected on their (001) and (010) surfaces. Notably, AHC is expected if these materials are slightly charge doped.

The crystal structure with the calculated magnetic ground state of Mn(Bi, Sb)₄Se₇ is shown in Figure 1a. The space group of the primitive cell is $P\bar{3}m1$ (No. 164). It consists of a Mn(Bi, Sb)₂Se₄ septuple layer (SL) and a (Bi, Sb)₂Se₃ quintuple layer (QL), stacked by van der Waals interaction along *z* direction. In the paramagnetic state, the magnetic space group is $P\bar{3}m1'$ (No. 164.86) with the group generators as follows:

$$G = \{\hat{E}, \hat{I}, \hat{C}_{3z}, \hat{M}_x\} \otimes \{\hat{E}, \hat{T}\} \quad (1)$$

where \hat{E} stands for the identity operation, \hat{I} for the inversion operation, \hat{C}_{3z} for the 3-fold rotation around *z* axis, \hat{M}_x for the mirror operation about *x* = 0 plane and \hat{T} for time-reversal operation. In the interlayer antiferromagnetic state (A-AFM), the magnetic space group is $P_2\bar{3}c1$ (No. 165.96) with the generators as follows:

$$G = S \cup \{\hat{T}|\tau_{1/2}\}S \quad (2)$$

where

$$S = \{\hat{E}, \hat{I}, \hat{C}_{3z}, \hat{M}_x\tau_{1/2}\} \quad (3)$$

When magnetized along *z* direction (FM_{*z*}), the magnetic space group is $P\bar{3}m'1$ (No. 164.89) with the generators as follows:

$$G = S \cup \{\hat{T}|\hat{M}_x\}S \quad (4)$$

where

$$S = \{\hat{E}, \hat{I}, \hat{C}_{3z}\} \quad (5)$$

When magnetized along *x* direction (FM_{*x*}), the magnetic space group is $C2/m$ (No. 12.58) with the generators as follows:

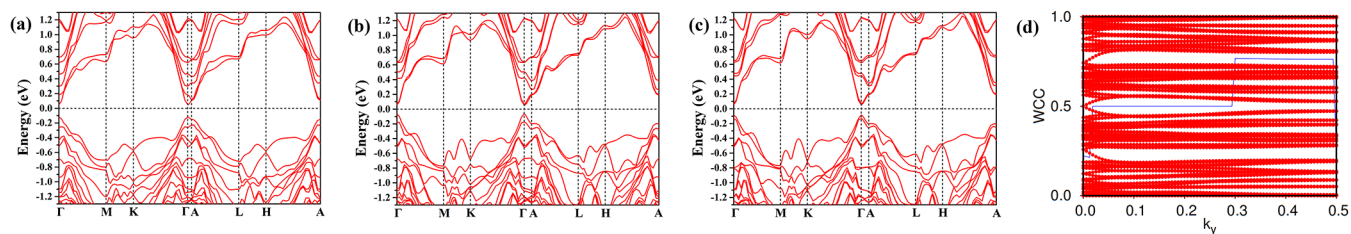


Figure 2. Band structures and Wilson loop of MnBi_4Se_7 . The band structure of A-AFM magnetic configuration (a), FMz magnetic configuration (b), FMx magnetic configuration (c), and the Wilson loop on the $k_z = 0$ plane of MnBi_4Se_7 A-AFM magnetic configuration (d).

$$G = \{\hat{E}, \hat{I}, \hat{M}_x\} \quad (6)$$

When magnetized along y direction (FM y), the magnetic space group is $C2'/m'$ (No. 12.62) with the generators as follows:

$$G = S \cup \{\hat{T}\hat{M}_x\}S \quad (7)$$

where

$$S = \{\hat{E}, \hat{I}\} \quad (8)$$

The computational details of this work can be found in Section II of [Supporting Information](#). In our calculations, the total energy of FMz is 0.08 meV/Mn lower than the FMx total energy in MnBi_4Se_7 . In MnSb_4Se_7 , this value is 0.06 meV/Mn. The total energy of FM y is nearly the same as the FMx total energy in our calculation. The A-AFM total energy is 0.09 meV/Mn lower than the FMz total energy in MnBi_4Se_7 . In MnSb_4Se_7 , this value is 0.10 meV/Mn. Based on these values, it seems that 0.032 and 0.035 T external magnetic field are needed to turn A-AFM state to FMz state for MnBi_4Se_7 and MnSb_4Se_7 , respectively. To turn the A-AFM state to the FMx state, 0.061 and 0.057 T external magnetic fields are needed for MnBi_4Se_7 and MnSb_4Se_7 , respectively.

However, it is well-known that the energy scale of the magnetic exchange energy and magnetic anisotropic energy in vdW materials are so small that it reach the accuracy limit of the DFT calculations. Thus, such estimated values may not be consistent with the experimental values. Such problems could be improved by using more accurate but very expensive density functionals, such as hybrid functionals or even GW calculations.

As a comparison, 0.15 T external magnetic field is needed to turn MnSb_4Te_7 from A-AFM ground state to FMz state in experiments. We thought the strength of external magnetic field to turn the magnetic states of MnBi_4Se_7 and MnSb_4Se_7 will not be too large compared to MnSb_4Te_7 since the nonmagnetic (Bi, Sb) $_2\text{Se}_3$ layers strongly weaken the interlayer magnetic exchange and the Mn 3d bands are slightly far away from the Fermi levels.

As is well-known, the topological properties and the behaviors of surface states are strongly dependent on the symmetries. Thus, there are evident opportunities of tunable magnetic topological properties in MnBi_4Se_7 and MnSb_4Se_7 . Since MnSb_4Se_7 has almost the same properties as those of MnBi_4Se_7 in our calculations, we choose MnBi_4Se_7 as an example for the following discussions.

As shown in [Figure 1b,c](#), the projected density of states (PDOS) of $\text{Mn}(\text{Bi}, \text{Sb})_4\text{Se}_7$ near the valence band maximum (VBM) and conduction band minimum (CBM) are mainly the p orbitals of Bi/Sb and Se, while the Mn 3d orbitals are slightly away from the VBM and CBM. The calculated bulk band structures of MnBi_4Se_7 with the A-AFM, FMz, FMx magnetic

orders are shown in [Figure 2a–c](#). All the above energy bands open a direct SOC band gap at the Γ point. As for the FM y case, the bulk energy band structure is very similar to the FMx although they have different magnetic space group symmetries.

In MnBi_4Se_7 , the bulk band gaps for A-AFM, FMz, and FMx are 167.2, 117, and 140.6 meV, respectively. We could understand the origin of the difference of the bulk band gap values in different magnetic configurations as follows. Since these bulk band gaps resulted from the SOC effect, they are quite sensitive to the magnetic configurations with different symmetries. Guaranteed by the $\hat{S} = \hat{T}\tau_{1/2}$ symmetry and inversion symmetry \hat{I} , all the bands are double degenerated as Kramer pairs with spin up and spin down states in the A-AFM configuration. When turned to the FM states, such Kramer degeneracy is lifted since the \hat{S} symmetry is broken down. Such band splitting may shift up the VBM and push down the CBM. Then the bulk band gap is reduced in FM states.

The pure spin magnetic moment is $5 \mu_B/\text{Mn}$ in $\text{Mn}(\text{Bi}, \text{Sb})_4\text{Se}_7$. However, the orbital magnetic moment can be different when the system is magnetized in different directions. In our calculations, the total magnetic moment is $4.9902 \mu_B/\text{Mn}$ for the FMz configuration and $4.9895 \mu_B/\text{Mn}$ for the FMx configuration in MnBi_4Se_7 . In MnSb_4Se_7 , this value is $4.9975 \mu_B/\text{Mn}$ for the FMz configuration and $4.9972 \mu_B/\text{Mn}$ for the FMx configuration. This suggests that the orbital magnetic moment in the FMz configuration is smaller than the orbital magnetic moment in the FMx configuration. This plays an important role in the SOC effect and affects the SOC bulk band gap.

In the A-AFM configuration of MnBi_4Se_7 , since the \hat{S} symmetry and inversion symmetry \hat{I} exist, the band topology of the A-AFM magnetic configuration can be diagnosed by the \mathbb{Z}_2 topological invariant:

$$(-1)^\lambda = \prod_{k_{\text{inv}}, n \in \text{occ}/2} \xi_n(k_{\text{inv}}) \quad (9)$$

where k_{inv} is the time-reversal invariant k points, n runs over all the occupied Kramers' pairs, and λ is the \mathbb{Z}_2 topological invariant. Topological nontrivial $\mathbb{Z}_2 = 1$ is obtained in the MnBi_4Se_7 A-AFM configuration, indicating that it is a topological insulator (TI) with quantum spin Hall channels existing on the \hat{S} -preserving surfaces. In order to further confirm this feature, we also calculate the Wannier charge center (WCC) on the half of $k_z = 0$ plane of the first Brillouin zone.^{51,52} As shown in [Figure 2d](#), the Wilson loop also indicates $\mathbb{Z}_2 = 1$. Since the system has the inversion symmetry, the band topology can also be described by the \mathbb{Z}_4 topological symmetry indicator:⁵³

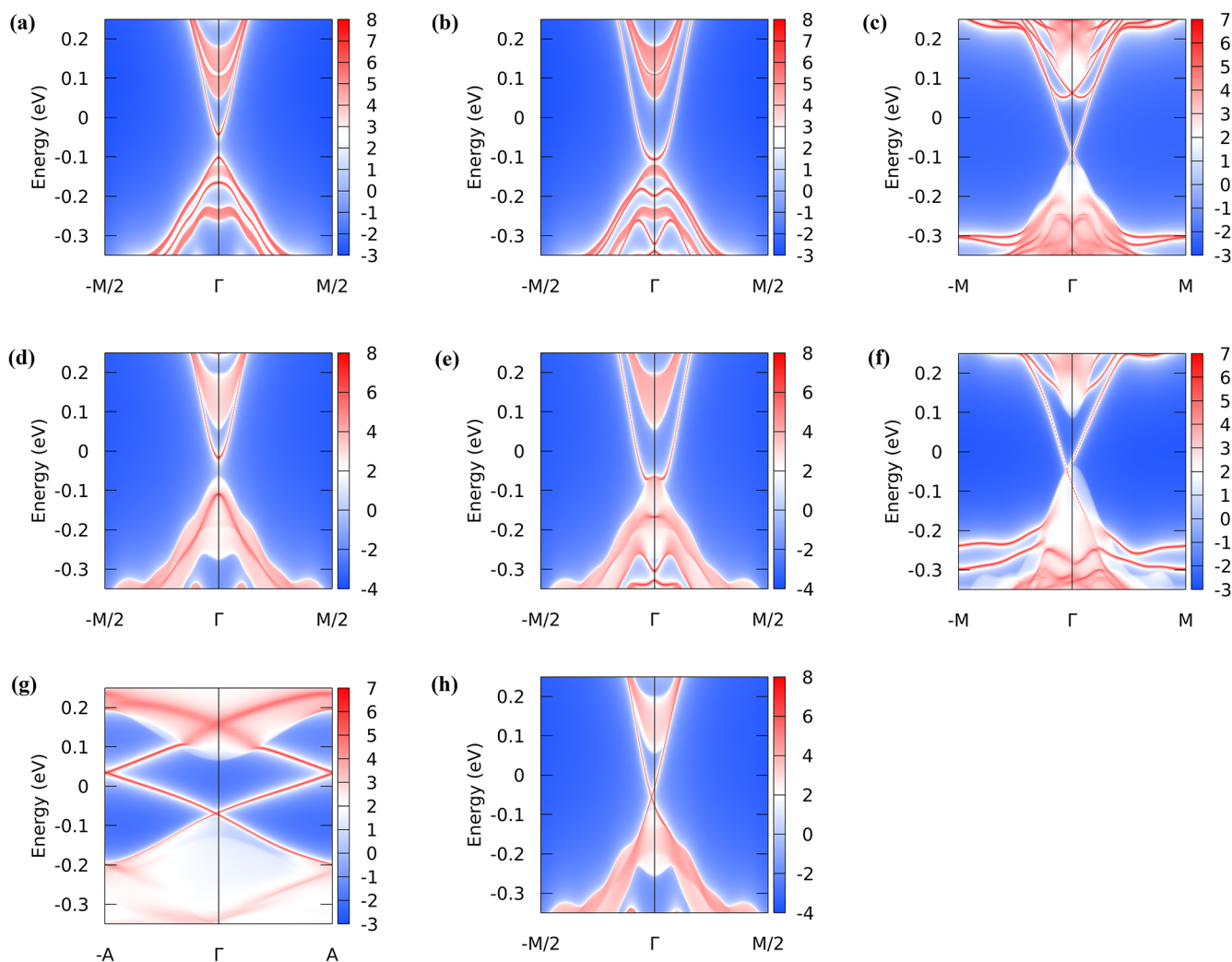


Figure 3. Surface states shown by LDOS confined to the surfaces of MnBi_4Se_7 . (a) The surface states on (001) surface's QL termination with A-AFM configuration, (b) the SL termination with A-AFM configuration, (c) the massless Dirac Fermion dispersion on the \hat{S} -preserving (110) surface with A-AFM configuration, (d) the surface states on (001) surface's QL termination with FMz configuration, (e) the SL termination with FMz configuration, (f) the surface states on (110) surface with FMz configuration, (g) the (010) surface with FMx configuration, and (h) the (001) surface's QL termination with FMx configuration.

$$\mathbb{Z}_4 = \sum_{\alpha=1}^8 \sum_{n=1}^{n_{\text{occ}}} \frac{1 + \xi_n(\Lambda_\alpha)}{2} \pmod{4}, \quad (10)$$

where Λ_α is the eight inversion invariant k points in the first Brillouin zone, n runs over all the occupied bands, and $\xi_n(\Lambda_\alpha)$ is the parity eigenvalue of the n th band at Λ_α . It is known that $\mathbb{Z}_4 = 1$ or 3 indicates the topological semimetal phase and $\mathbb{Z}_4 = 2$ indicates the axion insulator (AX) phase. In our calculations, like the MnSb_4Te_7 case,⁴³ $\mathbb{Z}_4 = 2$ is obtained no matter in AFM or FM configurations of MnBi_4Se_7 . Thus, quantized bulk orbital magnetoelectric coupling is expected in MnBi_4Se_7 materials.^{54,55}

The bulk-edge correspondence indicates that the nontrivial band topology and inverted band gap give rise to topological surface or hinge states on specific symmetry preserving surfaces or surface boundaries in 3-dimensional topological materials. As for the AFM topological insulators (TI), the gapless topological surface states exist on the \hat{S} -symmetry preserving surfaces. In $\text{Mn}(\text{Bi}, \text{Sb})_4\text{Se}_7$ systems, there are two different vdW gap terminations, namely the SL termination and QL

termination on the (001) surface. Based on the TB model, we have calculated the local density of states (LDOS) confined to specific surfaces using the iterative Green function method via the WannierTools package to show the surface state spectra.⁵⁶ In order to enhance the clarity of projected bulk states, the natural logarithm function is employed for the LDOS and thus negative values are tolerated. As shown in Figure 3a,b, the (001) surface state spectra from the $-M/2$ point (0, -0.25) to the $M/2$ point (0, 0.25) in the surface Brillouin zone are all gapped no matter the SL termination or the QL termination in the A-AFM configuration. However, as shown in Figure 3c, the massless Dirac Fermion dispersion is manifested clearly on the \hat{S} -preserving (110) surface. Thus, quantum spin Hall channels are expected theoretically on the \hat{S} -preserving surfaces in MnBi_4Se_7 of the A-AFM magnetic ground state.

When the system is magnetized to the ferromagnetic configurations, the \hat{S} symmetry is broken down. In the previous discussion, ferromagnetic MnBi_4Se_7 is an axion insulator with $\mathbb{Z}_4 = 2$. Generally speaking, the gapped surface state spectrum is more ideal for the experiments to observe the quantized bulk orbital magnetoelectric coupling. In our

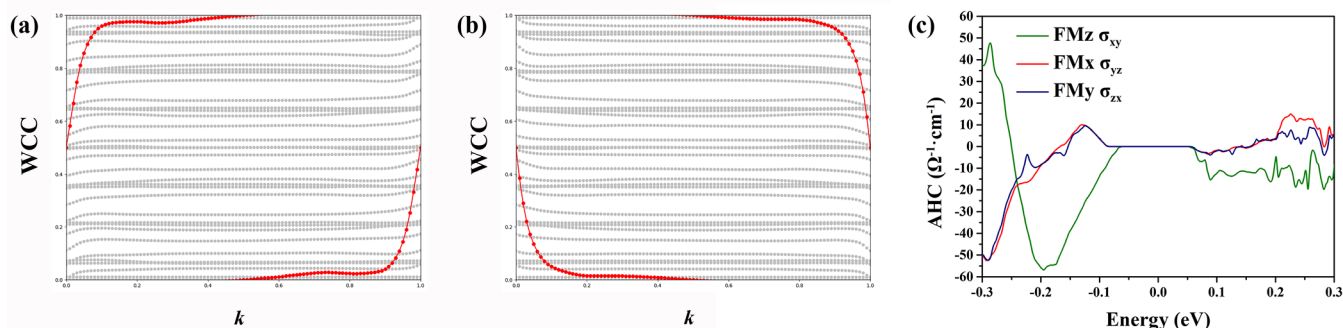


Figure 4. MCNs on the $k_x = 0$ plane with FM x configuration and AHC of MnBi₄Se₇. (a) The WCC with mirror eigenvalue $M_x = +i$, (b) the WCC with mirror eigenvalue $M_x = -i$, and (c) the AHC.

calculations, the surface state spectrum of MnBi₄Se₇ FM configurations is strongly dependent on the magnetization directions and the surface orientations. In the FMz configuration, as shown in Figure 3d,e, the calculated surface state spectrum is gapped on the QL termination on the (001) surface. However, on the SL termination, the surface state is overlapped with the projected bulk band structure and thus it is gapless. Lacking the \hat{S} symmetry, the surface state spectrum is still degenerated but the degenerate point shifts away from the Γ point on the (110) surface as shown in Figure 3f. It is found that the surface band structures are gapped on other surfaces, such as the (010) surface in the FMz state.

When turned to the in-plane magnetization, things are different. In the FM x configuration, $x = 0$ mirror symmetry exists and plays an important role in the topological property. The calculated SIs indicate that it is a topological crystalline insulator (TCI) with nontrivial mirror Chern numbers (MCNs) $n_{M_x=\pm i} = \pm 1$ defined on the $k_x = 0$ plane in the $+i$ and $-i$ mirror eigenvalue subspaces.⁵⁷ As shown in Figure 4(a)(b), the calculated flow of the WCC also confirms $n_{M_x=\pm i} = \pm 1$. Here, the (010) and (001) surfaces are mirror-invariant surfaces. As a consequence of the nontrivial MCNs, gapless surface states are found in their mirror-invariant k -paths on the $k_x = 0$ plane, as shown in Figure 3g,h.

Furthermore, we find that the magnetic topological phases of MnSb₄Se₇ are the same as those of MnBi₄Se₇ (see Section I of the Supporting Information). Meanwhile, its bulk band structures and surface states are also similar to that of MnBi₄Se₇ (see Figure S1 and Figure S2 in the Supporting Information). To better understand the similarities and differences of the MnBi₄Se₇, MnSb₄Se₇ in this work with the previously reported MnBi₄Te₇, MnSb₄Te₇, MnBi₂Se₄ and MnSb₂Se₄, we list their magnetic topological phases in 1 for

Table 1. Summary of Topological Phases for MnBi₄Se₇, MnSb₄Se₇, MnBi₄Te₇, MnSb₄Te₇, MnBi₂Se₄ and MnSb₂Se₄^a

compound	A-AFM	FMz	FMx
MnBi ₄ Se ₇	TI and AX	AX	TCI and AX
MnSb ₄ Se ₇	TI and AX	AX	TCI and AX
MnBi ₄ Te ₇	TI	unreported	unreported
MnSb ₄ Te ₇	TI and AX	AX	AX
MnBi ₂ Se ₄	TI	Weyl semimetal	Weyl semimetal
MnSb ₂ Se ₄	TI	Weyl semimetal	Weyl semimetal

^aTI, AX, and TCI stand for topological insulator, axion insulator, and topological crystalline insulator, respectively.

comparison. Although the TCI phase is not yet reported in MnBi₄Te₇ and MnSb₄Te₇, we believe this topological phase is possible in these two compounds since they have the same magnetic space group symmetry as the MnBi₄Se₇ and MnSb₄Se₇ in the FM x configuration.

Finally, we have calculated the AHC for both MnBi₄Se₇ and MnSb₄Se₇ in FMz, FM x , and FMy magnetic configurations by the method of Berry curvature.⁵⁸

$$\sigma_{xy} = -\frac{e^2}{\hbar} \int_{\text{BZ}} \frac{d^3k}{(2\pi)^3} \Omega^z(\mathbf{k}) \quad (11)$$

where $\Omega^z(\mathbf{k})$ is the Berry curvature at \mathbf{k} points. The integral runs over all the occupied states in the first Brillouin zone. As shown in Figure 4c and Figure S3c (see Supporting Information), no matter below or above the band gap, notable AHC is expected in these systems when they are magnetized to ferromagnetic states. Such situation is similar to that of the MnSb₄Te₇ case. Since the synthesized MnSb₄Te₇ sample is slightly hole doped and several Weyl points exist near the Fermi energy, large AHC is measured experimentally by Guo et al. Although no any Weyl node with opposite chirality ± 1 is observed between the conduction band and valence band near the Fermi energy in MnBi₄Se₇ and MnSb₄Se₇, the AHC values are still notable in the charge doped metallic states of these two compounds. Furthermore, since intrinsic defects widely exist in the experimental samples of this material family,⁵⁹ such result is also possible in Mn(Bi,Sb)₄Se₇ systems. Another way to achieve the charge doping is the gate voltage, which is also a powerful way to drive the magnetic phase transitions experimentally.^{60,61}

To draw a conclusion, we have predicted that both MnBi₄Se₇ and MnSb₄Se₇ are magnetic topological material candidates via first-principles calculations. In the A-AFM magnetic ground state, both are simultaneously antiferromagnetic topological insulators and axion insulators. Massless Dirac Fermion dispersions exist on their \hat{S} -symmetry preserving surfaces. When magnetized to ferromagnetic states, they are axion insulators. Furthermore, they are also topological crystalline insulators in the FM x configuration. Mirror-symmetry-protected gapless surface states emerge on their (010) and (001) surfaces. Notable AHC could be expected when these compounds are charge doped. We expect that our work will provide more opportunities for the research of magnetic topological physics.

■ ASSOCIATED CONTENT

Data Availability Statement

The data and input files for DFT calculations and source codes for the MCN calculations that support the findings of this study are available upon reasonable request from the authors.

SI Supporting Information

The Supporting Information is available free of charge at <https://pubs.acs.org/doi/10.1021/acs.jpcllett.3c00162>.

Magnetic topological phases of MnSb_4Se_7 and computational details (PDF)

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Notes

The authors declare no competing financial interest.

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